ELSEVIER

Physica B 240 (1997) 289–297

PHYSICA B

Potential distribution in a finite 1-D array of arbitrary mesoscopic tunnel junctions

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Received 2 September 1997

Abstract

A general expression for the distribution of the potential in a finite one-dimensional array of arbitrary mesoscopic tunnel junctions in terms of Green's function is deduced. It is shown that the so-called partial "solitary" problem of mesoscopic tunnel junctions can be formulated similarly to the problem of the behavior of a electron in an one-dimensional tight binding and in a set of random delta-function models. We calculate analytically the potential distribution of a finite chain, taking into account the details of the geometry, particularly, the finite-size distribution of the metallic islands. It is shown that the knowledge of the potential distribution allows us, in principle, to calculate the capacitance and by this the diameter of the islands as an isolated sphere. This reveals to have its significance for design strategies of single electron devices particularly if chemically size tailored metal nanoparticles are used as building units.

Keywords: Cluster; Nanostructures; Green's function; Single-electron tunneling

1. Introduction

Advances in chemistry allow the synthesis of single-shape metal nanoparticles, the so-called ligand stabilized metal clusters, consisting of a well-defined number of metal atoms surrounded and by this stabilized by an organic ligand shell [1, 2]. As proposed recently, this metal clusters can be used to build up a new generation of single electron devices [3], consisting of a limited and well defined number N of clusters separated by the thickness of the dielectric ligand shells. Herein the metallic core and its surrounding insulating shell replace the islands and tunnel junctions, respectively, by conventional single-electron tunneling (SET) arrangements.

In one-dimensional arrays of mesoscopic tunnel junctions time and space correlations between tunneling events may appear due to the Coulomb blockade effect [4, 5]. The time oscillations of correlated single-electron tunneling events, i.e. the so-called charge soliton, can be counted very precisely and can be used in practice for possible metrological applications as well as for digital devices [6–9]. These arrays are usually fabricated by lithographic techniques in which the typical size of the junction is a few tens of nanometers. If chemical nanostructures are used to build up these kind of devices one tends to a length scale of 1–2 nanometers, where deviations in the number of atoms, which form the metal cluster core, influence its size and shape and therefore the form of the charge soliton.

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PII S0921-4526(97)00447-X

Originally, the charge-soliton propagation, would take place in a homogeneous 1-D and infinitely long array of identical junctions, where the potential distribution does not change its form, when an electron tunnels from one electrode to another [4, 5, 10–13] This means that in an infinite chain approximation the potential profile $\varphi_i^{(0)}$ does not depend on the number N of junctions in the array. On the other hand, in some special devices like the "SET-turnstile" and the "SET-pump" [6, 7], the soliton can extend over approx. 6–10 tunnel junctions and therefore enters the same scale as the array itself. In this situation the main condition of the infinite array approach, i.e. $-N \ln \Pi \gg 1$ (see Eq. (4)), is not satisfied. Then the finite number of junctions becomes crucial and starts to play an important role in the formation of the potential profile.

The limits of validity of the infinite chain approximation have been discussed in the recent papers by Hu and O'Connell [14, 15]. They found an exact solution for the charge soliton in the case of a finite 1-D array of N gated junctions with equal junction capacitances C and equal gate capacitances C_0 and as well as for a single-electron multi-junction trap. The key to the approach was, on the one hand, to adopt the semiclassical model to describe the 1-D array [10] and, on the other hand, to rewrite the charge conservation law and Kirchhoff's laws as equations for the island potentials $\{\varphi_i^{(0)}\}$, instead of the island voltages $\{V_i\}$.

The purpose of this paper is to consider a finite 1-D array consisting of N small metal clusters with a finite-size distribution, which are arranged in series (see Fig. 1), and to find an exact analytical solution for the potential distribution in terms of Green's function (GF). The GF approach enables us to formulate the so-called partial "solitary" problem of small mesoscopic tunnel junctions similar to the problem of the behavior of an electron in an 1-D tight binding and in a set of random delta-function models. Applying this to the consideration of metal-cluster arrays, the following discussion will be performed with the assumption that the capacitance C is the same for all junctions whereas the self-capacitance C_0 can fluctuate from site to site due to a finite size distribution. Further, we assume that the metal clusters have a continuous density of states, i.e. we exclude quantum size effects and its influence on the capacitance from our consideration. Generally, this simplifications are not necessary assumptions for the method discussed here.

The paper is organized as follows. The GF of the tight-binding model with a class of periodic Hamiltonians will be considered and is introduced in Section 2. This method is a convenient way to describe the influence of substituting impurities onto the energy spectrum in a perfect periodic lattice. In the course of our consideration impurity has the meaning of size variation and therefore deviation of capacitance from the original value C_0 along the

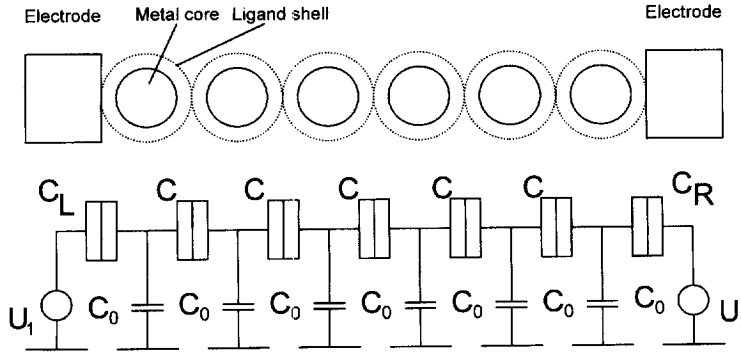

Fig. 1. Schematic drawing of $N = 6$ small metal clusters, fabricated in series and separated by the thickness of the dielectric ligand shells. The capacitance C corresponds to the thickness of the ligand shells and is the same for all junctions. The self-capacitance C_0, which corresponds to the cluster size, can fluctuate from site to site due to a finite size distribution.

linear chain of junctions. Some well-known results concerning one and two impurities in terms of GF are recalled in Section 3. In Section 4, we obtain the analytical expression for the potential profile for a finite 1-D array with two impurities in the case of asymmetric biased electrodes.

2. Method of calculation

Before calculating the GF for a finite 1-D array, we briefly explain the model of the infinite 1-D array approach, where the soliton-like solution can be described in the framework of the tridiagonal model [10] as well as the tight-binding model with a class of periodic Hamiltonians. In the tridiagonal model for a 1-D array of N junctions the only nonzero elements are the diagonal elements $C_{i,i} = C_0$, which is representing the self-capacitance of the islands, and the nearest-neighbor elements $C_{i,i\pm1} = C$, dominated by the tunnel-junction capacitance.

The potential $\varphi_i^{(0)}$ can be described by the following set of linear equations, which follow from the charge conservation law

$$-C\varphi_{i-1}^{(0)} + (2C + C_0)\varphi_i^{(0)} - C\varphi_{i+1}^{(0)} = Q_i \quad \text{for} \quad i = 1, \ldots, N-1. \tag{1}$$

Here we assume that we know the electric charge $Q_i = en_i$ of all electrodes.

The partial "solitary" or single-charge soliton solution of Eq. (1), where there is no charge on any of the islands except that a single charge appears on the kth island, i.e.

$$Q_i = e\delta_{i,k} \tag{2}$$

can be calculated explicitly. The potential of an arbitrary island i as a function of the distance from the charged kth island and very far from the edges of the array is given by [10]

$$-\varphi_i^{(0)} = \frac{e}{C_{\text{eff}}}\Pi^{|i-k|}. \tag{3}$$

Here $\Pi \equiv (x - \sqrt{x^2-1})$, $x = 1 + C_0/2C$ and $C_{\text{eff}} = \sqrt{C_0^2 + 4CC_0}$. We note that the Eq. (3) is correct in the limit

$$-N \ln \Pi \gg 1, \tag{4}$$

i.e. very far from the end of the array.

As for the differential equation for the matrix GF, $G(i,k)$ for the periodic tight-binding Hamiltonian operator in the case of a 1-D infinite linear chain discretized into a lattice can be written in the form [16]

$$-VG_{i-1}^{(0)} + (2V + \varepsilon_0)G_i^{(0)} - VG_{i+1}^{(0)} - EG_i^{(0)} = \delta_{i,j}. \tag{5}$$

Here we denote the off-diagonal matrix element $\varepsilon_{i,i\pm1}$ of the periodic tight-binding Hamiltonian by V and the diagonal matrix element $\varepsilon_{i,i}$ by $\varepsilon_0 + 2V$. E is the energy and the indices i, k denote points on a discrete lattice.

The GF for this class of periodic potentials can be calculated exactly [16] and we have

$$G^{(0)}(i,k;E) = \frac{1}{\sqrt{(E-\varepsilon_0)^2 - 4V^2}} \times [y - \sqrt{y^2-1}]^{|i-k|}, \tag{6}$$

where $y = (E - \varepsilon_0)/2V$ and the upper index (0) of GF indicates that the GF is calculated in the case when the system is infinite.

Formally, the corresponding GF $G^{(0)}(i,k;0)$ and therefore the potential $\varphi_i^{(0)}$ of 1-D Poisson's equation for the infinite chain of mesoscopic tunnel junctions can be obtained from the Eq. (6) by substituting $E = 0$, $V = C$ and $\varepsilon_0 = -(C_0 + 2C)$. Then one has (in the following the parameter $E = 0$ will be omitted in the argument of Green's function)

$$G^{(0)}(i,k) = \frac{\varphi_i^{(0)}}{e}, \tag{7}$$

where the $\varphi_i^{(0)}$ is given by Eq. (3).

This kind of proportionality between the potential $\varphi_i^{(0)}$ and GF follows directly also from the general solution of Poisson's equation expressed in terms of the GF of the whole system through the integral relation:

$$\varphi(x) = -4\pi \int G(x,x')\rho(x')\,dx'. \tag{8}$$

Really, as following from the integral equation Eq. (8) with the assumption (2), as well as in the case of discretized spatial coordinate, when the Green's function becomes a matrix $G(i, k)$, the potential $\varphi_i \sim G(i, k)$.

So the calculation of the potential distribution for the realistic geometry of a finite 1-D array is reduced to the calculation of the Green's function for the whole system, taking into account all boundary effects.

To achieve this goal it is convenient to adopt the method of contact Green's function developed in Refs. [17, 18]. This method has been applied to various problems in solid-state physics before, when the system considered is size limited, e.g. in solids containing interfaces between different defined crystals [19–21]. In this approach the first step is the evaluation of GF between two semi-infinite media, assuming that we know the GF of each medium. In other words, the problem is formulated in such a way as to be able to solve it with allowance for one interaction and then taking into account a second interaction without thereby making any assumptions. Therefore, the problem iteratively with arbitrary $N + 1$ boundaries can be solved, considering the solution with N boundaries to be known.

For the further consideration we introduce the expression for the GF of an isolated and good metallic conductor, i.e. the left (right) electrode is large enough to be considered as unbounded with a self-capacitance $C_L(C_R)$. The corresponding expression for the diagonal GF $G_i^{(0)}(i, i)$ $(i = L, R)$ of the left (right) electrode follows directly from the Eq. (7) under the condition that $C_{L,R} \gg C$

$$G_i^{(0)}(i, i) = -\frac{1}{C_i}. \tag{9}$$

Note that if the $C_{L,R} \to \infty$ then $G_L^{(0)}(L, L)$ $(G_R^{(0)}(R, R))$ tends to zero and this is the case of an array with unbiased edges.

Before ending this section, we note that in the recent paper by Likharev and Matsuoka [22], using the continuum limit approach, in which the discrete periodic structure is replaced by a continous dielectric medium, it was shown that in the three-dimensional electron–electron interaction problem the spatial distribution of the electrostatic potential of a long-linear array without a conducting ground plane does not depend strongly on the details of the geometry of its islands. Eq. (5) of Ref. [22], whose numerical integration yields authors to the main results of the paper cited, can directly be obtained from the Eq. (8) of this paper by integration with respect to the two-dimensional wave vector (Hankel's transform).

3. Infinitely long array with one, two and N impurities

To derive exact expressions for the potential distribution with many impurities labeled $l_j > l_{j-1}$ and $j = 1, \ldots, N$, we start from the periodic array and modify the self capacitance in each site step by step.

Let us start from the case when the 1-D perfect periodic array of capacitance has been modified at just one site (e.g. the l_1) and the diagonal self-capacitance $\tilde{C}_{l_1}$ equals $C_0 + C_1$. C_1 can be negative or positive. In the course of consideration of a cluster array one cluster is replaced by a smaller or larger one, respectively, at site l_1. The complete GF, which allows us to take into account this modification at the site l_1, related to the unperturbed $G_0^{(0)}(i, k)$ (Eq. (7)) and takes the form [16, 23]

$$G^{(1)}(i, k) = G^{(0)}(i, k) + r_1 \frac{G^{(0)}(i, l_1) G^{(0)}(l_1, k)}{G^{(0)}(l_1, l_1)}, \quad -\infty \leqslant i, k \leqslant +\infty, \tag{10}$$

where

$$r_1 = \frac{C_1 G^{(0)}(l_1, l_1)}{1 - C_1 G^{(0)}(l_1, l_1)}. \tag{11}$$

The quantity r_1 is well known from the theory of tight binding and a set of delta function models [16, 23]. This quantity, which is the complex amplitude of the reflection of an electron from the single impurity at a given site is model dependent. For both, the tight-binding model and a set of delta functions, we obtain

$$r_l = \frac{V_l G^{(0)}}{1 + V_l G^{(0)}}, \tag{12}$$

where V_l is the lth diagonal energy in the tight-binding case, and the strength of the lth delta function in the other case. $G^{(0)}(l, l)$ is the unperturbed GF for each case.

Although, in general, the real transport of electrons in arrays is incoherent, nevertheless, a similar formulation can be used for the soliton problem and the quantity r_1 can be considered as the reflection amplitude of a soliton passing the capacitance $\tilde{C}_{l_1}$. If C_1, and therefore r_1 is zero we have a homogeneous system without any soliton's reflection.

Using Eqs. (7) and (10) we can rewrite the potential of an arbitrary electrode i as a function of the distance from the charged kth electrode in the form

$$\varphi_i^{(1)} = \varphi_i^{(0)}(1 + r_1 \Pi^{|i-l_1|+|l_1-k|-|i-k|}), \tag{13}$$

where $\varphi_i^{(0)}$ and r_1 are given by Eqs. (3) and (11), respectively.

By use of Eq. (13) we examine the influence of one impurity on the potential distribution in an infinite 1-D array. This is illustrated in Fig. 2 for a negative value of $C_1/C_0 = -0.999$, which means that we decrease the size and by this the self-capacitance of the island in l_1 (we choose this relation just for better graphical illustration). At large distances, the function (Eq. (13)) approach the unperturbed single-electron soliton distribution (Eq. (3)) while at small distances we have a second peak which is located at site l_1. In the reverse case (not shown in Fig. 2), which means that the change of capacitance is positive, i.e. the size of the island is increased, we find a dip at site l_1. Equipped with this result, we regard again a chain of ligand stabilized metal clusters. In this chain the self-capacitance C_0 will directly be proportional to the diameter of the metal core, if it is regarded as a sphere. Therefore, the potential distribution in both cases leads directly to the size of the modified cluster at site l_1, or reverse, the size of the cluster leads us directly to the potential profile of the chain.

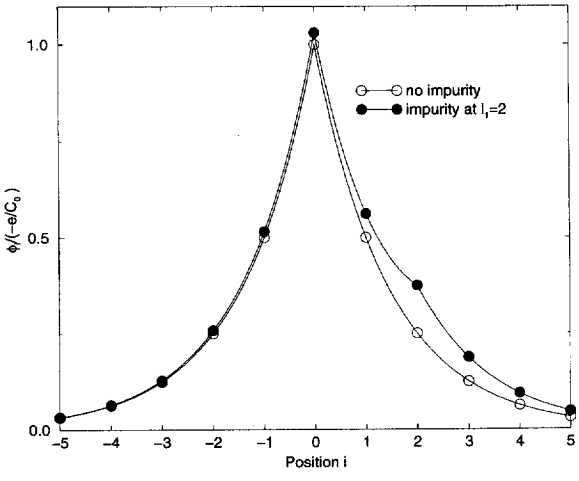

Fig. 2. Potential distribution of the infinite array in the units of $-e/C_0$ and for one impurity at $l_1 = 2$ with $C_1/C_0 = -0.999$.

Now, let us modify second self-capacitance from the right, i.e. at the site l_2. The diagonal capacitance $\tilde{C}_{l_2}$ equals $C_0 + C_2$ and as in the case of the first impurity C_2 can be negative or positive. The complete GF, which allows us to take into account this substituting of the host capacitance at the l_2 site, and related to the unperturbed $G_0^{(0)}(i, k)$, Eq. (7) can be written in the form [16, 23]

$$G^{(2)}(i, k) = G^{(1)}(i, k) + R_2^{\rightarrow}\frac{G^{(1)}(i, l_2)G^{(1)}(l_2, k)}{G^{(1)}(l_2, l_2)},$$

$$l_1 \leqslant i, k \leqslant l_2, \tag{14}$$

where

$$R_2^{\rightarrow} = \frac{C_2 G^{(1)}(l_2, l_2)}{1 - C_2 G^{(1)}(l_2, l_2)} = \frac{r_2(1 + r_1\lambda_{2,1})}{1 - r_1 r_2 \lambda_{2,1}}, \tag{15}$$

where $R_2^{\rightarrow}$ is the amplitude of the soliton's reflection from 2nd center; the arrow indicates the direction of the soliton's propagation. The quantity $R_2^{\rightarrow}$ differs from r_1 since it included also the impurity at the point l_2. The $G^{(1)}(i, k)$ is given by Eq. (10).

The quantity $\lambda_{2,1}$ in Eq. (15) is given by

$$\lambda_{2,1} = \lambda_{1,2} = \frac{G^{(0)}(l_2, l_1)G^{(0)}(l_1, l_2)}{G^{(0)}(l_2, l_2)G^{(0)}(l_1, l_1)} = \Pi^{2(l_2 - l_1)}. \tag{16}$$

Here we use the explicit expression given by Eq. (7) for the $G^{(0)}(i, k)$.

On the other hand, the expression for $G^{(2)}(i, k)$ for $i, k \geqslant l_2$ in terms of the bare GF $G^{(0)}(i, k)$ can be

written in the form

$$G^{(2)}(i,k) = G^{(0)}(i,k) + R_2^{\leftarrow}\frac{G^{(0)}(i,l_2)G^{(0)}(l_2,k)}{G^{(0)}(l_2,l_2)}$$

$$\equiv G^{(0)}(i,k)(1 + R_2^{\leftarrow}\Pi^{i+k-2l_2-|i-k|}) \qquad (17)$$

The unknown quantity $R_2^{\leftarrow}$ can be obtained from the condition of continuity of the new GF Eq. (17) and GF Eq. (14) at the point $i = l_2$. It is easy to verify that the soliton's reflection amplitude from the right of two impurities is

$$R_2^{\leftarrow} = \frac{r_2 + r_1(1+2r_2)\lambda_{2,1}}{1 - r_1 r_2 \lambda_{2,1}}. \qquad (18)$$

The expression for $G^{(2)}(i,k)$ on the left side of two impurities ($i,k \leqslant l_1$) will be similar to the Eq. (17) and can be obtained from Eq. (17) by substituting $l_2 \to l_1$ in the expression $G^{(0)}(i,k)$ and by changing the direction of propagation in Eq. (18) with $2 \to 1$.

$$G^{(2)}(i,k) = G^{(0)}(i,k) + R_1^{\rightarrow}\frac{G^{(0)}(i,l_1)G^{(0)}(l_1,k)}{G^{(0)}(l_1,l_1)}$$

$$\equiv G^{(0)}(i,k)(1 + R_1^{\rightarrow}\Pi^{2l_1-i-k-|i-k|}), \qquad (19)$$

$$R_1^{\rightarrow} = \frac{r_1 + r_2(1+2r_1)\lambda_{2,1}}{1 - r_1 r_2 \lambda_{2,1}} \qquad (20)$$

The expression for the GF in the case, when we have $i \leqslant l_1 \leqslant k \leqslant l_2$ ($i \leqslant l_1 \leqslant l_2 \leqslant k$) we can find using the relation connected electron's GF for spatial coordinate $G(x,x')$ with the diagonal GF at the coinciding coordinates $x = x'$ for one-dimensional systems [24]

$$G(x,x') = [G(x,x)\,G(x',x')]^{1/2}$$

$$\times \exp\left\{-\int_{\min(x,x')}^{\max(x,x')}\frac{dz}{2G(z,z)}\right\}. \qquad (21)$$

Following this relationship, it leads us to the expression of the GF

$$G^{(2)}(i,k) = G^{(0)}(i,k)\frac{(1+r_1)(1+r_2\lambda_{k,2})}{1 - r_1 r_2 \lambda_{1,2}},$$

$$i \leqslant l_1 \leqslant k \leqslant l_2. \qquad (22)$$

Finally, for the GF in the intervals ($i \leqslant l_1$ and $l_2 \leqslant k$) we have

$$G^{(2)}(i,k) = G^{(0)}(i,k)\frac{(1+r_1)(1+r_2)}{1 - r_1 r_2 \lambda_{1,2}}. \qquad (23)$$

By using the Eqs. (14), (17), (22) and (23) we have the explicit form for the potential distribution over the whole range of the array. We examine first the case where both C_1 and C_2 are negative. The analysis is of the behavior of the potential shows that again the negative change of the self-capacitance at both sites leads to peaks correspondingly. The bigger the change of the self-capacitance the more pronounced the peak.

Adding now a further impurity at the site l_3 and having written the condition of continuity for the new GF at the point $i = l_3$, we can evaluate the GF in the region ($l_2 \leqslant i,k \leqslant l_3$) and the amplitude of reflection on the left $R_3^{\leftarrow}$ and right $R_3^{\rightarrow}$ of the new boundary. Repeating this procedure N times we can obtain the potential distribution $\varphi_i^{(N)}$ for an arbitrary interval of a whole system, which was our intention.

As we see from our discussions the potential distribution φ_i of the whole system is defined by the potential of each interval of interest. In a given interval ($i \leqslant l_1$ and $l_N \leqslant k$) the potential $\varphi_i^{(N)}$ can be presented in the form

$$-\varphi_i^{(N)} = \frac{e}{C_{\text{eff}} D_N}\Pi^{k-i}, \qquad (24)$$

where the D_N is the determinant of a tridiagonal matrix $N \times N$. The matrix elements of D_N is given by

$$(D_N)_{nl} = \delta_{nl} + \frac{C_l}{C_{\text{eff}}}\Pi^{|n-l|} \qquad (25)$$

and it satisfies the following recurrence relationship

$$D_l = A_l D_{l-1} - B_l D_{l-2}. \qquad (26)$$

The index l goes from 1 to the number of impurities (islands) N and the initial conditions are:

$$A_1 = 1 + \frac{C_1}{C_{\text{eff}}}, \qquad D_0 = 1, \qquad D_{-1} = 0. \qquad (27)$$

We have for $l > 1$:

$$A_l = 1 + B_l + \frac{C_l}{C_{\text{eff}}}(1 - \Pi^2) \tag{28}$$

and

$$B_l = \frac{C_l}{C_{l-1}}\Pi^2. \tag{29}$$

Let us consider the case when in an infinite 1-D array we have a segment with N identical self-capacitances, e.g. $\bar{C}_l = C_0 + \bar{C}$. Then the recurrence relationship (Eq. (26)) can be solved exactly for D_N and Eq. (24) for the potential distribution $\varphi_i^{(N)}$ can be rewritten as follows (e.g. at $i = 0, k = N$):

$$-\varphi_0^{(N)} = \frac{e}{C_{\text{eff}}}\left\{\cosh(N\beta) + \left(\frac{\bar{C}}{C_{\text{eff}}}\cosh(\ln \Pi) - \sinh(\ln \Pi)\right) \times \frac{\sinh(N\beta)}{\sinh(\beta)}\right\}^{-1}, \tag{30}$$

where

$$\cosh(\beta) = \cosh(\ln \Pi) - \frac{\bar{C}}{C_{\text{eff}}}\sinh(\ln \Pi). \tag{31}$$

We note that Eq. (31) is the analogous of the electron's energy spectrum equation for the 1-D Kronig–Penney chain of identical and negative potentials, rewritten for the negative value of energy.

4. Finite array with impurities

All expressions given by Eqs. (14), (17), (22) and (23) for the GF were found in the case of an infinite array. In a real situation, i.e., in practice, the edge electrodes of a finite system are biased with different voltages. This means that in the case of a finite chain the GF should be supplemented by these boundary conditions. To do this we start to consider the situation of two semi-infinite conductors with general boundary conditions. We shall assume that the left-hand half-space I, occupying the region $i, k \leqslant R$, is characterized by the GF Eq. (17) and the right-hand half-space II in the region $i, k \geqslant R$ is represented by the GF Eq. (9).

Using the method applied in Ref. [21] to the "cooperon" problem in disordered metal films, we can present the GF $G^{(\mathrm{I})}(i,k)$ in the left-hand half-space I in the form

$$G^{(\mathrm{I})}(i,k) = G^{(2)}(i,k) - r_{12}\frac{G^{(2)}(i,R)G^{(2)}(R,k)}{G^{(2)}(R,R)}, \quad l_2 \leqslant i, k \leqslant R, \tag{32}$$

and $G^{(2)}(i,k)$ is given by Eq. (17). This is the expression for the potential distribution of left-hand half-infinite array with two impurities at the sites l_1, l_2. The quantity r_{12} is the relaxation amplitude for a soliton passing from the region I into region II and it can be written in the form

$$r_{12} = -r_{21} = \frac{G^{(2)}(R,R) - G_R^{(0)}(R,R)}{G^{(2)}(R,R) + G_R^{(0)}(R,R)}. \tag{33}$$

The first term on the right-hand side of the potential distribution of the half-infinite array corresponds to the direct propagation and the second term to reflection from the boundary. In the limit $C_R \to \infty$ (the region I is in a contact with the unbiased edge) we find that $r_{12} \to 1$. According to Eq. (32), this means that $G^{(\mathrm{I})}(i,k) \leqslant G^{(2)}(i,k)$, i.e. there is a decreased probability to return to the initial point, because the unbiased electrode can be considered to be a big reservoir. In other words, the unbiased edge of the array can be described as a mirror, where the soliton will interact with his mirror-image soliton (or anti-soliton) and therefore will be attracted [13].

The analogous expression for the GF $G^{(\mathrm{I})}(i,k)$ in the right-hand side of the biased left electrode (the left-hand half-space 0) with the capacitance C_L will be similar to the Eq. (32). This expression in the range $R \leqslant i, k \leqslant l_1$, can be obtained from Eq. (32) by substituting $R \to L$ and $2 \to 0$:

$$G^{(\mathrm{I})}(i,k) = G^{(2)}(i,k) - r_{10}\frac{G^{(2)}(i,L)G^{(2)}(L,k)}{G^{(2)}(L,L)}, \quad R \leqslant i, k \leqslant l_1, \tag{34}$$

and $G^{(2)}(i,k)$ is given by Eq. (19). The quantity r_{10} is the relaxation amplitude for a soliton passing from

the region I into region 0 and has the form

$$r_{10} = -r_{01} = \frac{G^{(2)}(L, L) - G_L^{(0)}(L, L)}{G^{(2)}(L, L) + G_L^{(0)}(L, L)}. \tag{35}$$

Finally, we show the expression for the $\varphi^{(I)}$ in the case when we have the left as well as the right electrode biased, two impurities and $l_1 \leqslant i, k \leqslant l_2$:

$$\varphi_i^{(I)} = \varphi_i^{(0)} D^{-1}(1 + R_I^{\leftarrow} R_I^{\rightarrow} \Pi^{2(l_2 - l_1)} - R_I^{\leftarrow} \Pi^{i+k-2l_1-|i-k|} - R_I^{\rightarrow} \Pi^{2l_2-i-k-|i-k|}), \tag{36}$$

where $\varphi_i^{(0)}$ is given by the Eq. (3). $R_I^{\leftarrow}$ is the amplitude of soliton reflection from the left block with one impurity at site l_1 and with one left electrode and it can be present in the form

$$R_I^{\leftarrow} = r_1 + r_{10} \frac{(1 + r_1)^2 \Pi^{2(l_1 - L)}}{1 + r_1^2 \Pi^{2(l_1 - L)}}. \tag{37}$$

$R_I^{\rightarrow}$ is the amplitude of soliton reflection from the right block with one impurity at site l_2 and one right electrode and it can be present in the form

$$R_I^{\rightarrow} = r_2 + r_{12} \frac{(1 + r_2)^2 \Pi^{2(R - l_2)}}{1 + r_2^2 \Pi^{2(R - l_2)}}. \tag{38}$$

The quantity $D^{(I)}$ has the form

$$D = 1 - R_I^{\leftarrow} R_I^{\rightarrow} \lambda_{1,2}. \tag{39}$$

It is straightforward to show that setting $R_I^{\rightarrow} = R_I^{\leftarrow} = 1$ ($r_1 = r_2 = 0$, $r_{10} = r_{10} = 1$) and $R = l_2$, $L = l_1$ in Eq. (36) we obtain the result of Ref. [14] for the single-charge soliton in a 1-D array of N gated junctions.

In Fig. 3, where we plot the potential distribution of a finite array with two impurities at the sites $l_1 = -2$ and $l_2 = 2$ illustrates the effect of two electrodes at its edges. The larger the spatial separation between the biased electrodes and the corresponding sites the smaller the influence onto the potential distribution. This means that the soliton solution Eq. (3) holds. But, even at large distances, i.e. in long chains of clusters with $L = -40$ and $R = 40$, we see a pronounced influence. Dependent on the bias of the electrode we can find a decrease of potential, which is maximal in the unbiased case, i.e. $r_{12} \to 1$ and $r_{10} \to 1$.

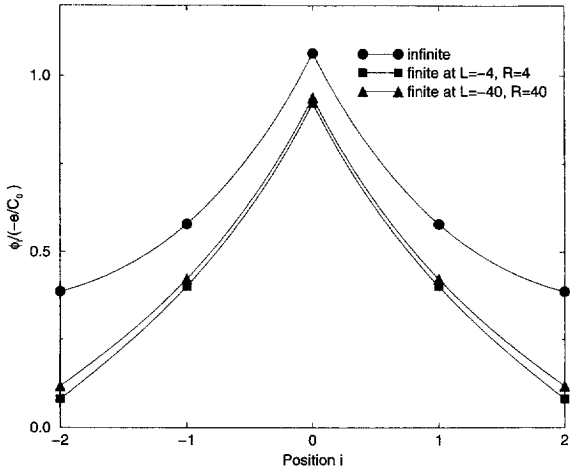

Fig. 3. The potential distribution of the finite array in the units of $-e/C_0$ and for two impurities at $l_1 = -2$, $l_2 = 2$ and different values for $L(= -R)$ with $C_1/C_0 = -0.999$.

5. Discussion

We developed a general expression for the distribution of the potential in a finite 1-D array of arbitrary mesoscopic tunnel junctions in terms of Green's function. This was applied to the consideration of a finite array consisting of ligand stabilized metal clusters. Herein the capacitance C, which corresponds to the inter cluster spacing, i.e. the thickness of the ligand shell, is kept constant, and the self-capacitance C_0, which corresponds to the cluster size, may change from site to site. The so-called partial "solitary" problem of mesoscopic tunnel junctions was formulated similarly to the problem of the behavior of an electron in a one-dimensional tight binding and in a set of random delta-function models. The potential distribution was calculated analytically for asymmetric biased electrodes, taking into account the size distribution of the islands along the array. It is shown that from the potential distribution the size variation of the spherical islands can be obtained directly. This reveals to have its significance for design strategies and quality control of single-electron devices, which are built up by chemically tailored metal clusters. By the knowledge of the influence of size distribution on the potential distribution, as described here, as well as on conductance peak

spacing [25], electrical characteristics of multijunction arrays may be foreknown.

Acknowledgements

We would like to thank M. Pollak, G. Schön and A.B. Zorin for helpful discussions, comments and a critical reading of the manuscript and L. Neubert for the computer calculations. Financial support by the Bundesminister für Bildung, Wissenschaft und Forschung (BMBF) under contract No. 03N1012A7 is gratefully acknowledged.

References

[1] L.J. De Jongh, in: G.C. Hadjipanayis, R.W. Siegel (Eds.), Nanophase Materials, Kluwer Academic Publishers, Dordrecht, 1994, p. 349.
[2] G. Schmid (Ed.), Clusters and Colloids, VCH, Weinheim, 1994.
[3] G. Schön, U. Simon, Colloid Polym. Sci. 273 (1995) 101 and 273 (1995) 202.
[4] K.K. Likharev, IBM J. Res. Dev. 32 (1988) 144.
[5] D.V. Averin, K.K. Likharev, in: B. Altshuler, P.A. Lee, R.A. Webb (Eds.), Mesoscopic Phenomena in Solids, Elsevier, Amsterdam, 1991, p. 173.
[6] D. Esteve in: H. Grabert, M.H. Devoret (Eds.), Single Charge Tunneling, Ch. 3, NATO ASI, Ser. B, Plenum, New York, 1992.
[7] D.V. Averin, K.K. Likharev, in: H. Grabert, M.H. Devoret (Eds.), Single Charge Tunneling, Ch. 9, NATO ASI, Ser. B, Plenum, New York, 1992.
[8] A.B. Zorin, Phys. Rev. Lett. 76 (1996) 4408.
[9] A.B. Zorin et al., Phys. Rev. B 53 (1996) 13682.
[10] K.K. Likharev, N.S. Bakhalov, S.G. Kazacha, S.I. Serdyukova, IEEE Trans. Magn. 25 (1989) 1436.
[11] L.S. Kuzmin, P. Delsing, T. Cleason, K.K. Likharev, Phys. Rev. Lett. 62 (1989) 2539.
[12] M. Amman, E. Ben-Jacob, K. Mullen, Phys. Lett. A 142 (1989) 431.
[13] P. Delsing, in: H. Grabert, M.H. Devoret (Eds.), Single Charge Tunneling, NATO ASI, Ser. B, Plenum, New York, 1992, p. 249.
[14] G.Y. Hu, R.F. O'Connel, Phys. Rev. B 49 (1994) 16773.
[15] G.Y. Hu, R.F. O'Connel, Phys. Rev. Lett. 74 (1995) 1839.
[16] E.N. Economou, in: P. Fulde (Ed.), Green's Functions in Quantum Physics, Springer, Berlin, 1983.
[17] F. García-Moliner, J. Rubio, J. Phys. C 2 (1969) 1789.
[18] B. Velicky, I. Bartoš, J. Phys. C 4 (1971) L104.
[19] F. García-Moliner, F. Flores, Introduction to the Theory of Solid Surfaces, Cambridge University Press, Cambridge, 1979, p. 100.
[20] H. Ueba, S.G. Davison, J. Phys. C 13 (1980) 1175.
[21] V.M. Gasparian, A.G. Aronov, B.L. Altshuler, Sov. Phys. Solid State 29 (1987) 1538.
[22] K.K. Likharev, K.A. Matsuoka, Appl. Phys. Lett. 67 (1995) 3037.
[23] V.M. Gasparian, Sov. Phys. Solid State 31 (1989) 266.
[24] A.G. Aronov, V.M. Gasparian, U. Gummich, J. Phys.: Condens. Matter 3 (1991) 3023.
[25] S.T. Ruggiero, J.B. Barner, Z. Phys. B 85 (1996) 333.